

A Brief Introduction to Control of Distributed Parameter Systems in Engineering Practice

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At present in the control of distributed parameter systems
there are two basic approaches

Early lumping approach - based on discretization of controlled objects

Late lumping approach – based on results of theory of partial differential equations

Here presented approach is based on the concept of **nonlinear lumped input and distributed parameter output system** - NLDS



About motivation – nowadays in engineering practice are widely used numerical models for dynamical analysis of technological and production (T&P) processes

Where changes of output fields of T&P processes $YN(\mathbf{x},t)$ are analysed in relation to changes of lumped manipulatable input quantities $\{U_i(t)\}_i$ as real NLDS

Based on numerical models of T&P processes in linearised surroundings of the steady-state regimes we can calculate distributed step and impulse responses

$$\left\{ \mathcal{H}H_i(\mathbf{x}, kT) \right\}_{i=1}^n \quad T = 1$$

$$\left\{ GH_i(\mathbf{x}, k) = \mathcal{H}H_i(\mathbf{x}, k) - \mathcal{H}H_i(\mathbf{x}, k-1) \right\}_{i=1}^n$$

for discrete convolution model

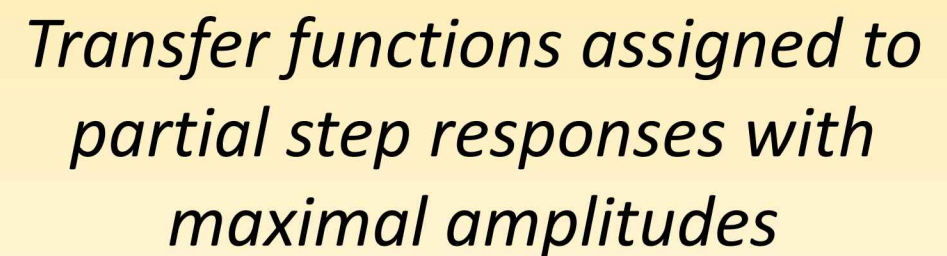
$$Y(\mathbf{x}, k) = \sum_{i=1}^n Y_i(\mathbf{x}, k) = \sum_{i=1}^n \sum_{q=0}^k GH_i(\mathbf{x}, k-q) U_i(q)$$

as linearized model of NLDS - **discrete lumped-input and distributed-parameter-output system with zero order hold units** - HLDS



where $Y(\mathbf{x}, k)$ is given as the summation of particular outputs $\left\{ Y_i(\mathbf{x}, k) \right\}_{i=1}^n$ corresponding to individual inputs $\left\{ U_i(k) \right\}_{i=1}^n$

based on distributed step responses $\left\{ \mathcal{H}_i(x, k) \right\}_{i=1}^n$

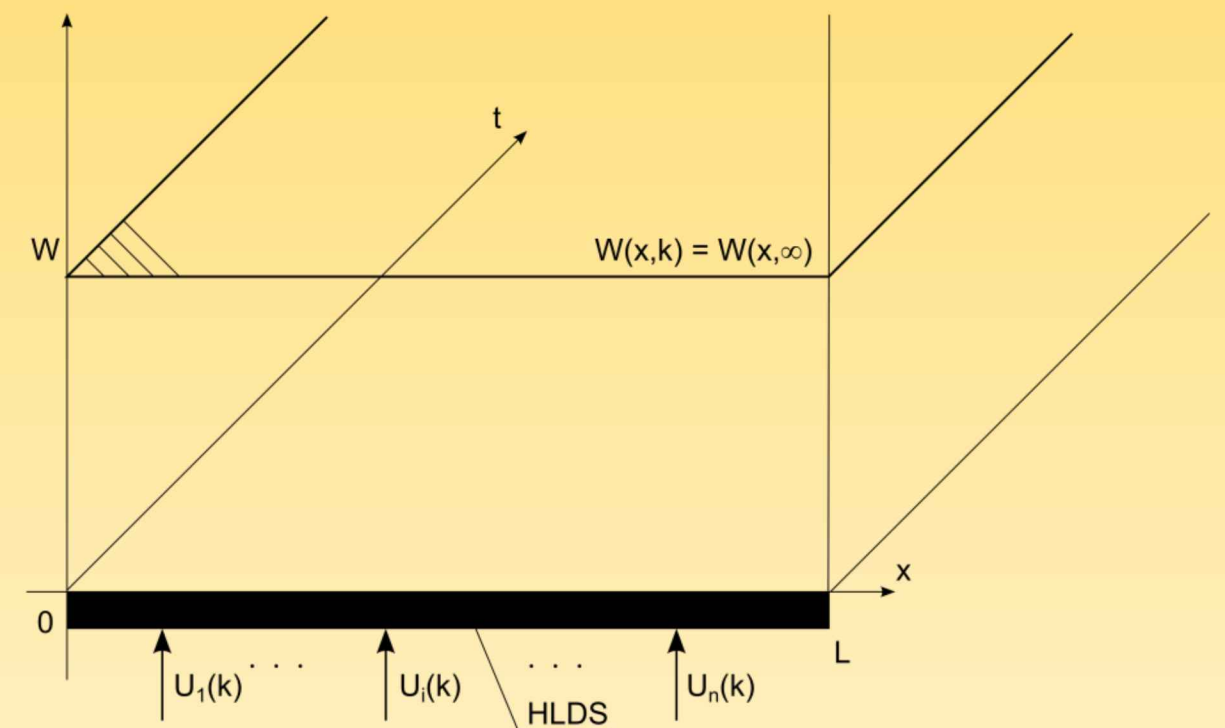
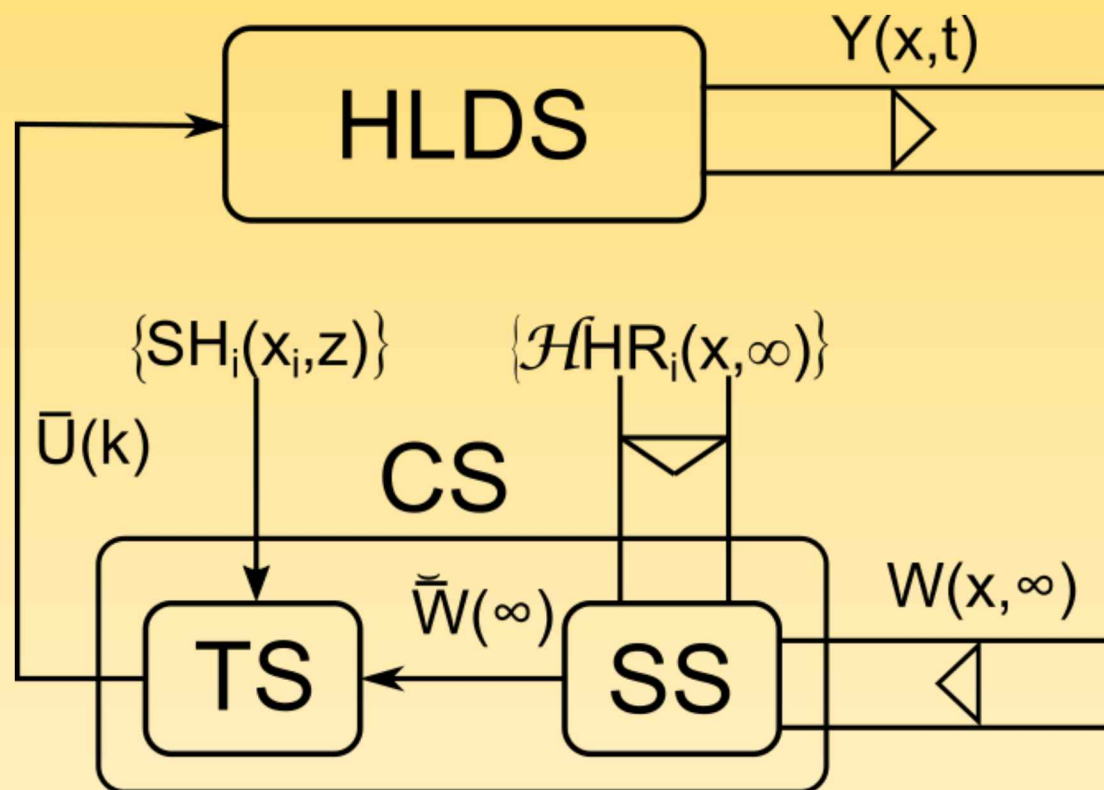

$$\left\{ \mathcal{H}R_i(x, \infty) = \frac{\mathcal{H}H_i(x, \infty)}{\mathcal{H}H_i(x_i, \infty)} \right\}_{i=1}^n$$

Time components

$$\left\{SH_i\left(x_i,z\right)\right\}_{i=1}^n$$

Open loop control of HLDS is solved from zero initial steady-state so that in the new steady-state control error will be minimal with vector of optimal actuating quantities $\bar{\bar{U}}(k)$

Distributed reference

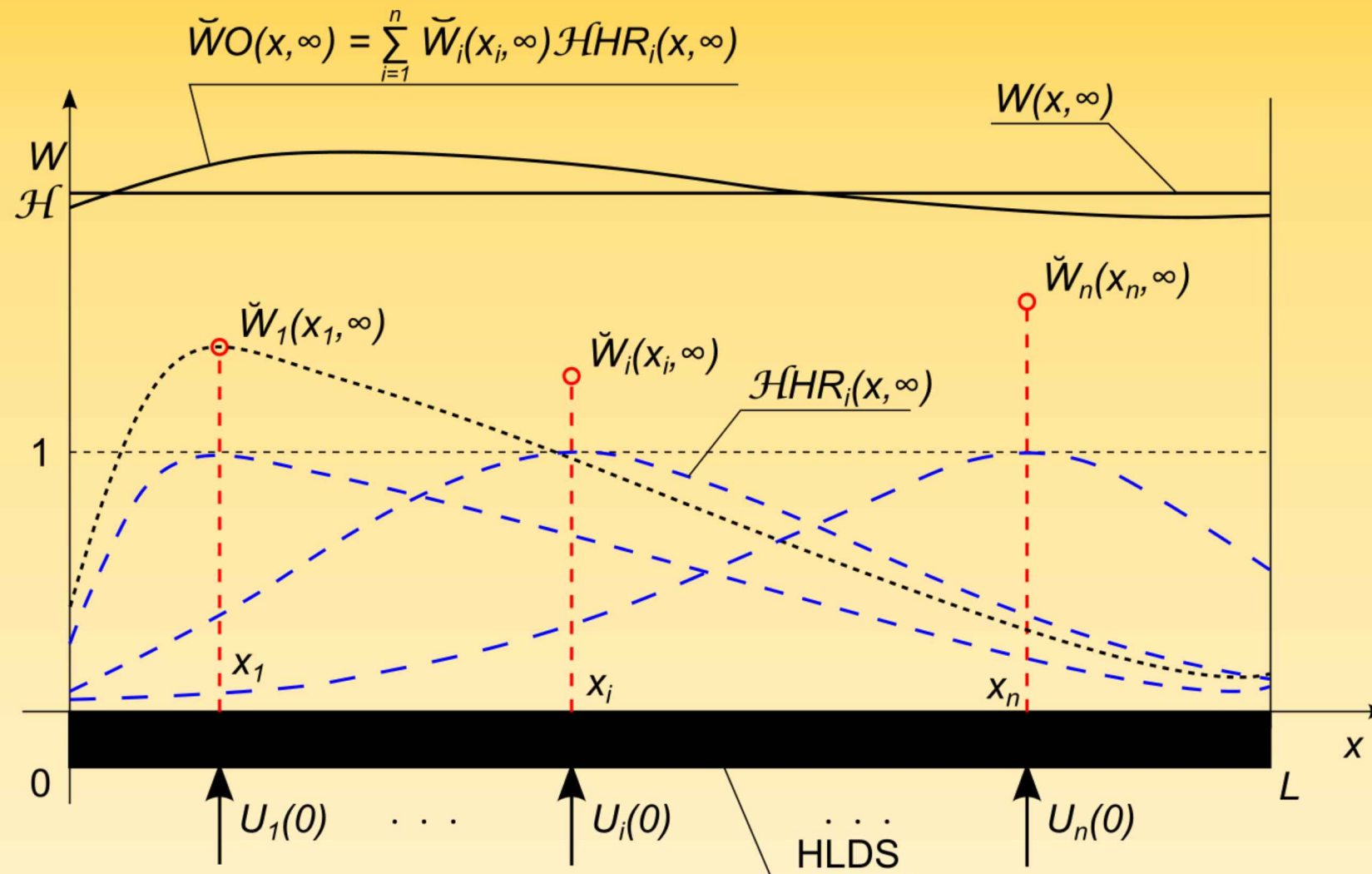


$$\bar{\bar{U}}(k) = ?$$

$$\min \|E(x, \infty)\|_2 = \min \|W(x, \infty) - Y(x, \infty)\|_2 = \|\bar{\bar{E}}(x, \infty)\|_2$$

CS, SS, TS – control synthesis, space synthesis, time synthesis blocks

Block SS – decomposition of distributed reference by approximation task on the set of space components of HLDS dynamics

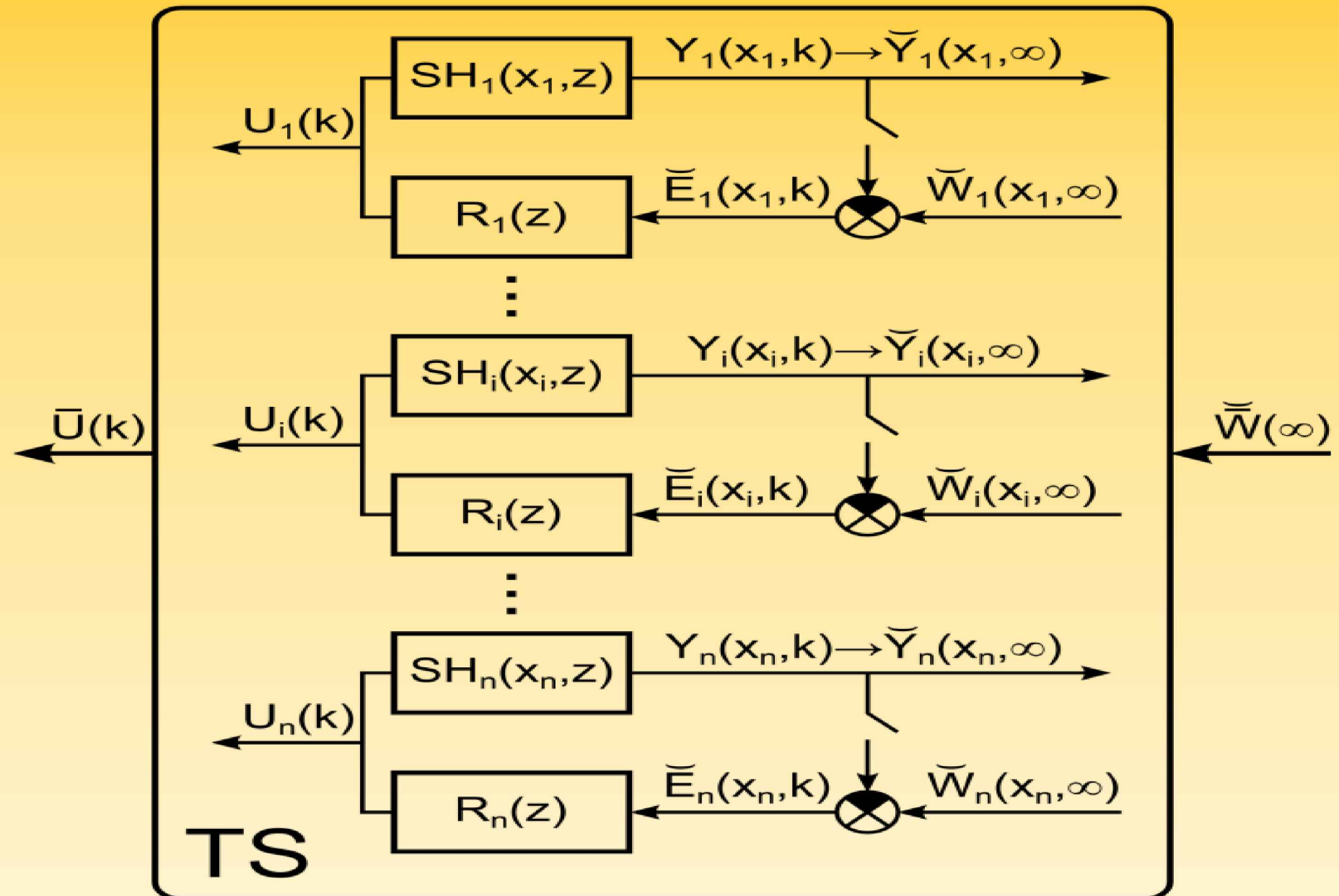


$$\min \left\| W(x, \infty) - \sum_{i=1}^n W_i(x_i, \infty) \mathcal{H}HR_i(x, \infty) \right\|_2 = \left\| W(x, \infty) - \sum_{i=1}^n \tilde{W}_i(x_i, \infty) \mathcal{H}HR_i(x, \infty) \right\|_2 = \left\| W(x, \infty) - \tilde{W}O(x, \infty) \right\|_2$$

$\tilde{W}O(x, \infty)$ is the unique best approximation of the $W(x, \infty)$ with minimal quadratic norm of their deviation

$\{\tilde{W}_i(x_i, \infty)\}_i = \tilde{\tilde{W}}(\infty)$ vector of optimal approximation parameters - output of the Block SS

Block TS – SISO control loops, where controlled systems are time components of HLDS dynamics, output is vector of actuating quantities

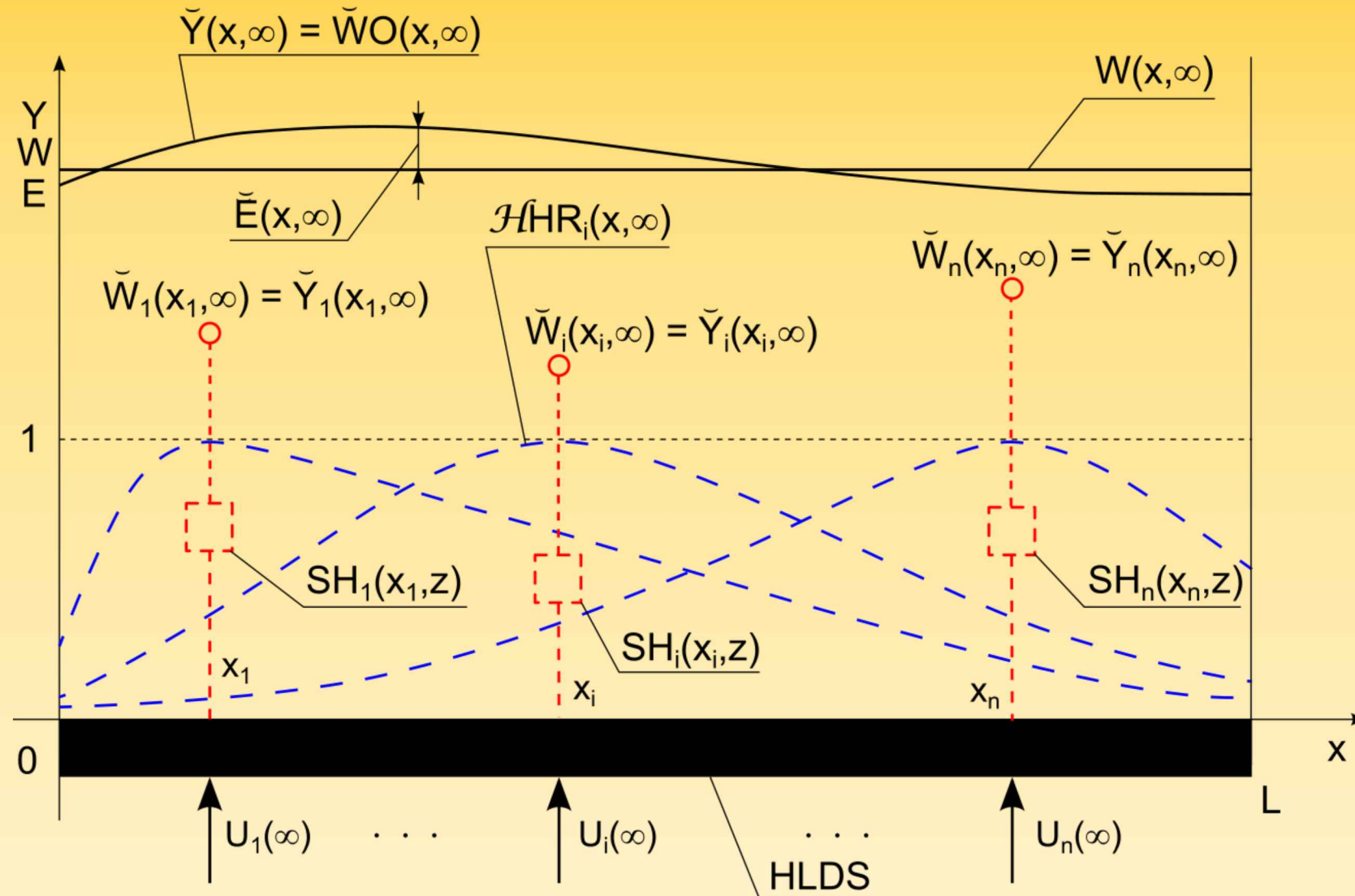


$$\left\{ \lim_{k \rightarrow \infty} \bar{E}_i(x_i, k) \right\}_{i,k} = \left\{ \lim_{k \rightarrow \infty} [\bar{W}_i(x_i, \infty) - Y_i(x_i, k)] = 0 \right\}_{i,k}$$

$$\{U_i(x_i, k)\}_i \rightarrow \{SH_i(x_i, z)\}_i \rightarrow \{Y_i(x_i, k) \rightarrow \bar{Y}_i(x_i, \infty) = \bar{W}_i(x_i, \infty)\}_i$$

partial outputs are equal to optimal approximation parameters in steady-state

Block HLDS – actuating quantities generate in new steady-state the unique best approximation of distributed reference through time components of HLDS dynamics along with partial outputs and space components of HLDS dynamics



$$\check{Y}(x, \infty) = \sum_{i=1}^n \check{Y}_i(x_i, \infty) \mathcal{H}HR_i(x, \infty) = \sum_{i=1}^n \check{W}_i(x_i, \infty) \mathcal{H}HR_i(x, \infty) = \check{W}O(x, \infty)$$

$\check{Y}(x, \infty)$ gives the unique best approximation of the $W(x, \infty)$

$$\lim_{k \rightarrow \infty} \|E(x, k)\|_2 = \|W(x, \infty) - \check{Y}(x, \infty)\|_2 = \|W(x, \infty) - \check{W}O(x, \infty)\|_2 = \|\check{E}(x, \infty)\|_2$$

then in the new steady-state control error is minimal

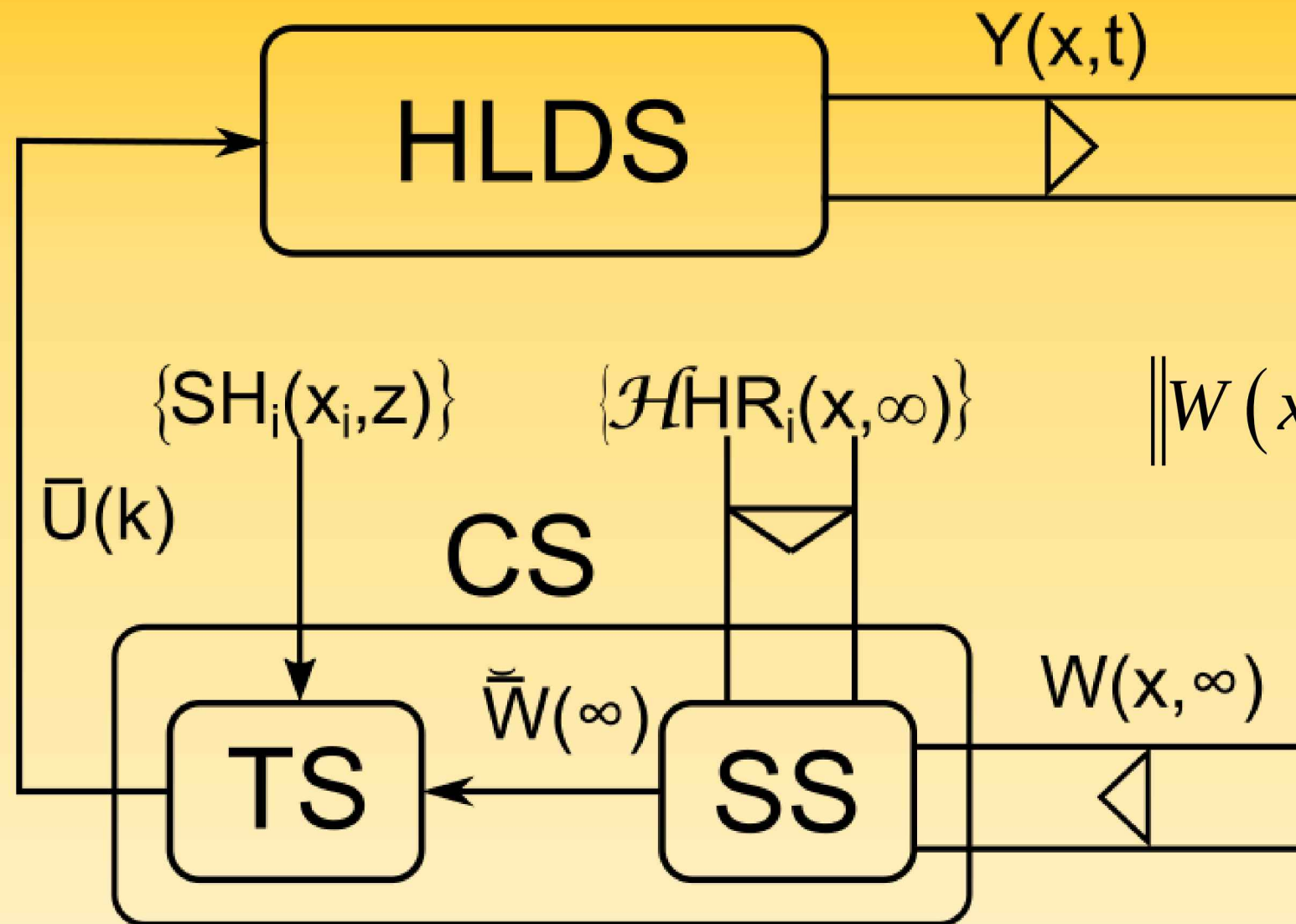
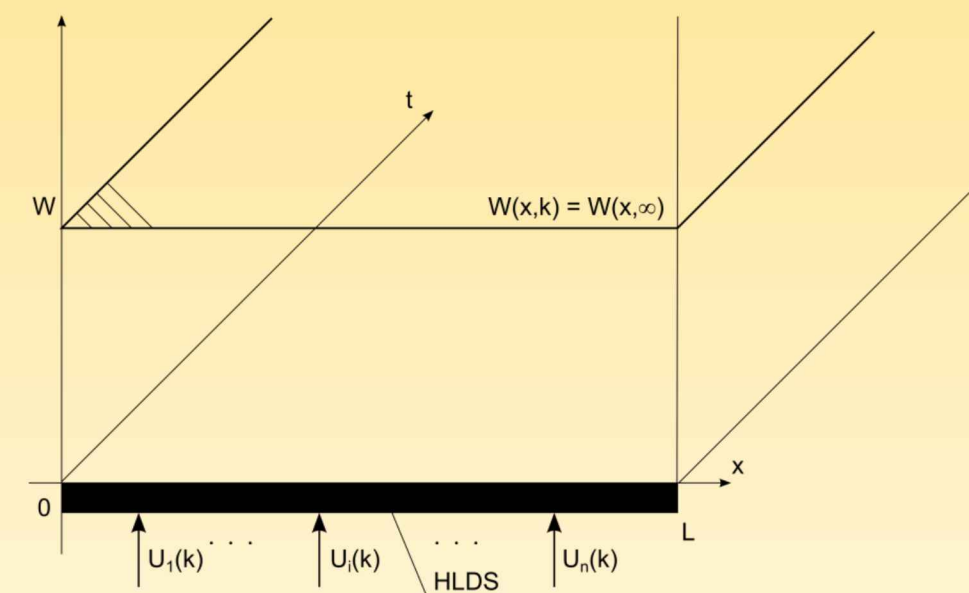
Open loop control of HLDS

$$\begin{aligned}\check{Y}(x, \infty) &= \sum_{i=1}^n \check{Y}_i(x_i, \infty) \mathcal{H}HR_i(x, \infty) = \\ &= \sum_{i=1}^n \check{W}_i(x_i, \infty) \mathcal{H}HR_i(x, \infty)\end{aligned}$$

Optimal output with minimal control error

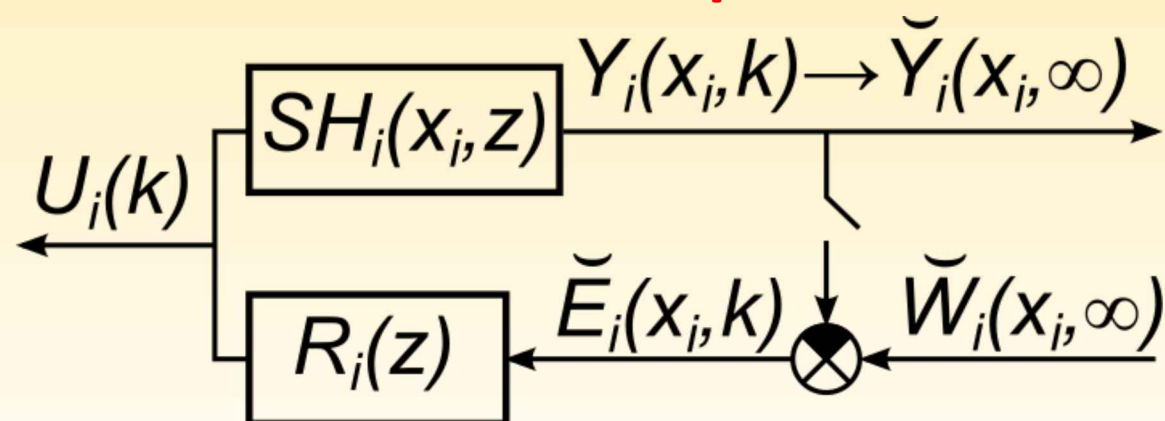
$$\|W(x, \infty) - \check{Y}(x, \infty)\|_2 = \|\check{E}(x, \infty)\|_2$$

Distributed reference



SISO control loops

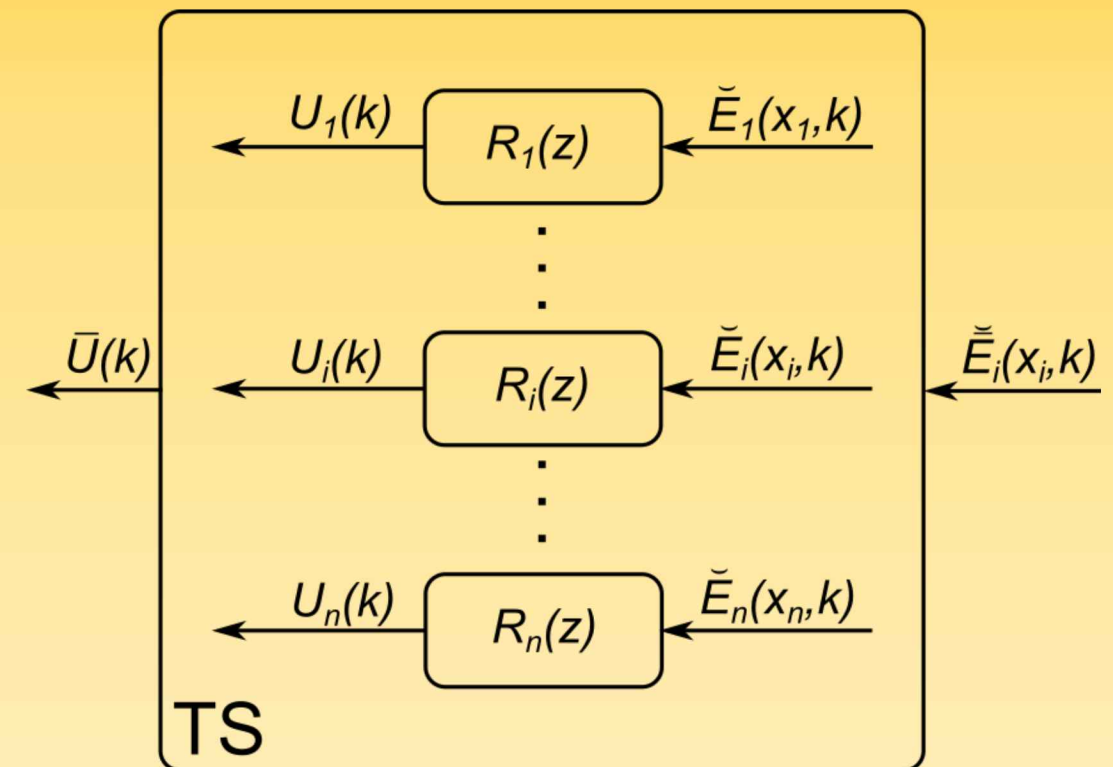
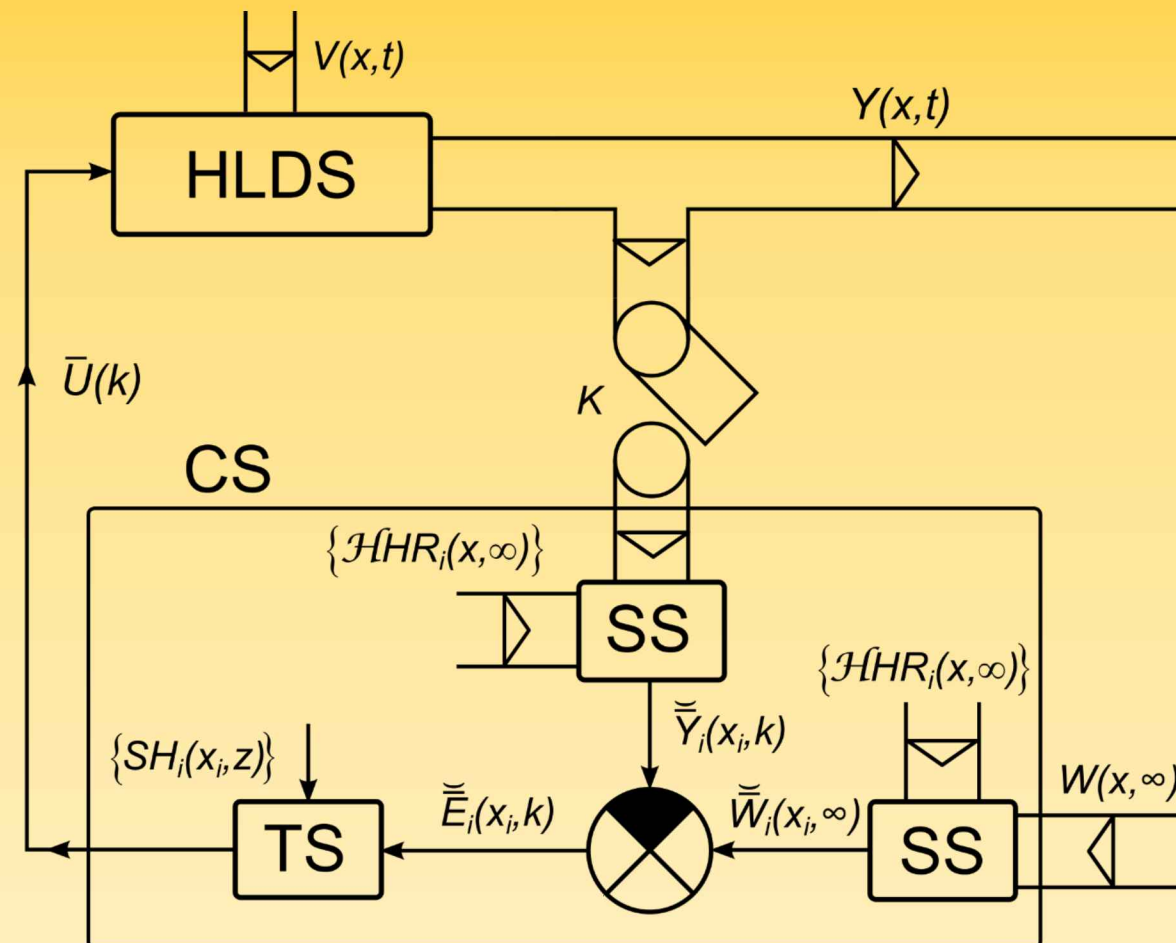
Decomposition



$$\min \left\| W(x, \infty) - \sum_{i=1}^n \check{W}_i(x_i, k) \mathcal{H}HR_i(x, \infty) \right\|_2$$

Control synthesis starts with unique best approximation and the same unique best approximation of the distributed reference there is also on the control loop output

Feedback control loop – based on introduced blocks SS and TS

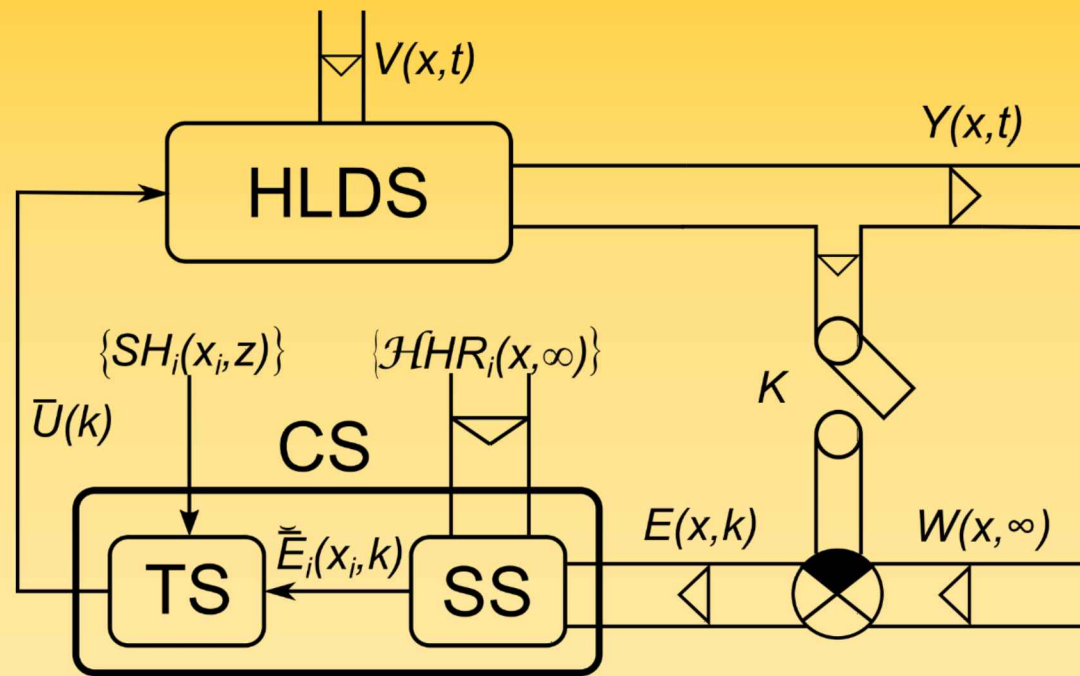


$$\min \left\| W(x, \infty) - \sum_{i=1}^n W_i(x_i, k) \mathcal{H}HR_i(x, \infty) \right\|_2 = \left\| W(x, \infty) - \sum_{i=1}^n \tilde{W}_i(x_i, k) \mathcal{H}HR_i(x, \infty) \right\|_2$$

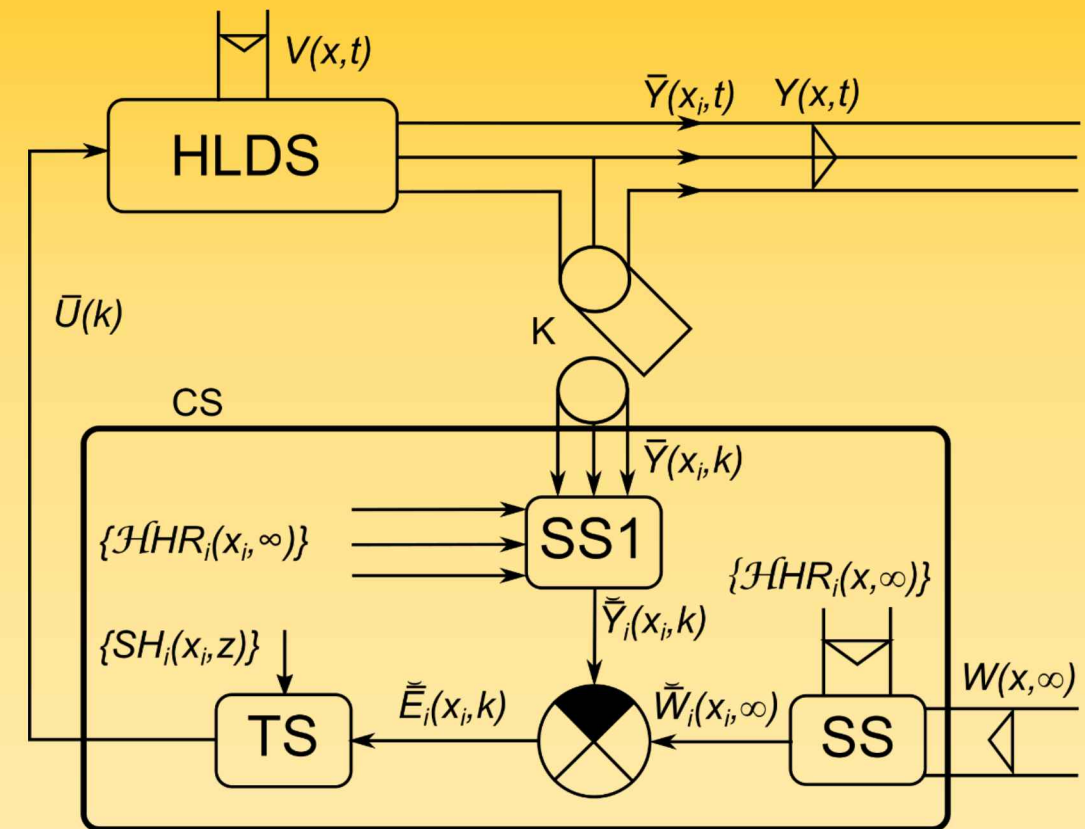
$$\min \left\| Y(x, k) - \sum_{i=1}^n Y_i(x_i, k) \mathcal{H}HR_i(x, \infty) \right\|_2 = \left\| Y(x, k) - \sum_{i=1}^n \tilde{Y}_i(x_i, k) \mathcal{H}HR_i(x, \infty) \right\|_2$$

Controllers in block TS - introduced in previous steps

Further feedback control loops



Control loop by merging two SS blocks from previous control loop



Control loop based on measurements in selected points of controlled objects

Further types of control loops based on reduced particular outputs and measurements in selected points of controlled objects are presented on the web site

www.dpscontrol.sk

Thank you for your attention.