

Control of Distributed Parameter Systems an Engineering Approach – Tutorial

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Abstract - The basic structure of an engineering approach for the control of distributed parameter systems will be presented in the scope of this tutorial. This method is based on numerical models of technological and manufacturing processes, represented as lumped input and distributed parameter output systems. Decomposition of dynamics and control synthesis into temporal and spatial components will be introduced along with distributed parameter open loop control and feedback control loops. Moreover, the basic software support called DPS Blockset for Simulink, a third-party software product of The MathWorks Company will be demonstrated along with some results of control of secondary cooling in continuous casting of steel.

Keywords - distributed parameter systems, nonlinear partial differential equations, lumped input and distributed parameter output systems, control systems, software support.

I. INTRODUCTION

Two principal approaches are used to solve engineering problems involving distributed parameter systems nowadays. The first is based on the discretization of plants - **early lumping approach**. The second is based on the results of partial differential equation theory along with the so-called **late lumping approach**.

The approach presented here is motivated by the availability of methods and software products for numerical analysis of technological and manufacturing processes. Such software products are widely used in modern engineering practice. The underlying dynamic numerical models are constructed by the numerical solution of non-linear partial differential equations describing these processes. Changes in the distributed output $Y(\mathbf{x}, t)$ of these systems are analyzed with the aid of these numerical models right on the level of distributed parameter states and outputs, while taking the individual inputs $\{U_i(t)\}$ into account (see Fig. 1.1). In essence, and from the viewpoint of the input/output relationship, these are **nonlinear lumped input and distributed parameter output systems** (NLDS), providing the basis of the presented approach.

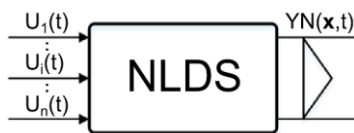


Fig. 1.1 Nonlinear lumped-input/distributed-output system.

Technological and manufacturing processes are usually operated in the vicinity of pre-determined steady-state

regimes where, under the action of suitable inputs $\{U_i(k)\}$ passed through zero order hold units, linearized changes are obtained on the output of NLDS. Thus, a linear lumped input and distributed parameter output system with zero order hold units (HLDS) is at disposal for the solution some control problems in the linearized surroundings of the given steady-states of NLDS, Fig. 1.2.



Fig. 1.2 Linear discrete lumped-input/distributed-output system with zero order hold units.

In the linearized surrounding of the given steady-state of the controlled NLDS the responses to unit step changes of inputs $\{U_i(k)\}$, through zero order holds, can then be computed by its numerical model as distributed parameter step responses $\{\mathcal{H}H_i(\mathbf{x}, kT)\}$, where T is sampling period. We may then also obtain discrete distributed parameter impulse responses given by $\{\mathcal{G}H_i(\mathbf{x}, kT) = \mathcal{H}H_i(\mathbf{x}, kT) - \mathcal{H}H_i(\mathbf{x}, (k-1)T)\}$ through subtracting the time shifted distributed parameter step responses for the discrete distributed parameter model

$$Y(\mathbf{x}, kT) = \sum_{i=1}^n Y_i(\mathbf{x}, kT) = \sum_{i=1}^n \sum_{q=0}^k \mathcal{G}H_i(\mathbf{x}, (k-q)T) U_i(qT) \quad (1.1)$$

with the so-called particular distributed parameter outputs $\{Y_i(\mathbf{x}, kT)\}$, where $\mathbf{x} = (x, y, z)$. This relation for a given T represents an input/output discrete model of HLDS for the solution some control problems in the surroundings of the given steady-states of NLDS, [1-4, 11].

II. DYNAMICS DECOMPOSITION

The spatial distribution of NLDS will be considered only in a one-dimensional interval $[0, L]$ for the clarity of presentation, however, the forthcoming results are also valid in three dimensions, $\mathbf{x} = (x, y, z)$. The particular distributed parameter step responses $\{\mathcal{H}H_i(\mathbf{x}, kT)\}$, for $T=1$, are computed in a linearized surrounding of a given steady-state as responses to unit step changes $\{U_i(k)=1(k)\}$, through zero order hold units, $\{U_i(t)=1(t)\}$, (see Fig. 2.1).

We may express the reduced distributed parameter step responses in the steady-state of $\{\mathcal{H}H_i(x, k)\}$ for $i = 1, n$ by

$$\left\{ \mathcal{H}HR_i(x, \infty) = \frac{\mathcal{H}H_i(x, \infty)}{\mathcal{H}H_i(x_i, \infty)} \right\} \quad (2.1)$$

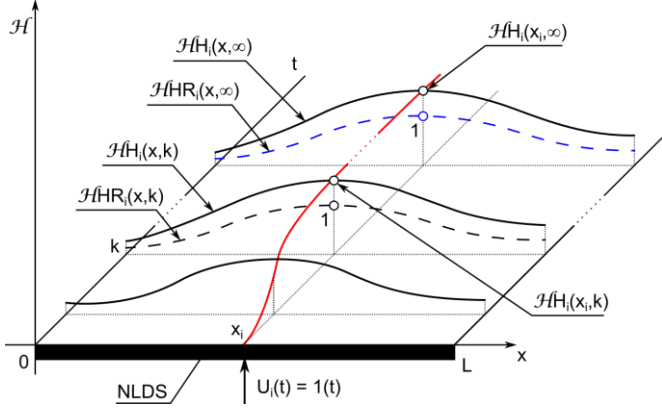


Fig. 2.1 i-th distributed parameter transient characteristic.

for $\{\mathcal{H}H_i(x_i, \infty) \neq 0\}$ in $\{x_i\}$, where $\{\mathcal{H}H_i(x, k)\}$ reach their maximal amplitudes in steady-states $\{\mathcal{H}H_i(x_i, \infty)\}$. Discrete transfer functions may be then identified and assigned to the partial distributed parameter step responses $\{\mathcal{H}H_i(x_i, k)\}$ as $\{SH_i(x_i, z)\}$ for $i = 1, n$.

Let us introduce the following particular temporal and spatial components of HLDS dynamics:

Temporal components

$\{SH_i(x_i, z)\}$ - for given i and chosen x_i - variable z

Spatial components

$\{\mathcal{H}HR_i(x, \infty)\}$ - for given i , $t \rightarrow \infty$, variable x

And finally for the steady-state of the HLDS we have

$$Y(x, \infty) = \sum_{i=1}^n Y_i(x, \infty) = \sum_{i=1}^n Y_i(x_i, \infty) \mathcal{H}HR_i(x, \infty) \quad (2.2)$$

III. OPEN LOOP CONTROL

Let us consider open loop control in the linearized neighborhood of the steady-state operating mode of the controlled plant. In order to simplify our discussion, the initial steady-state of this control loop will be considered zero and the control problem will be discussed as control of the linear HLDS, (see Fig. 3.1).

Let us consider a step change of the distributed parameter reference $W(x, k) = W(x, \infty)$ according to the discussion presented above (see Fig. 3.2). Furthermore, let the distributed parameter control synthesis be defined as finding the sequence of inputs $\bar{U}(k) = \{U_i(k)\}$ such, that the

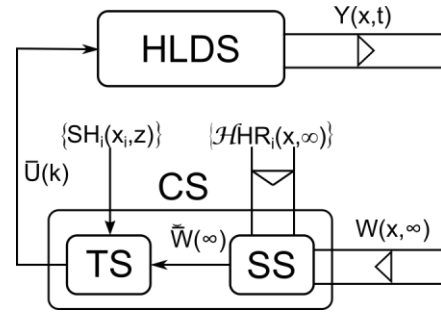


Fig. 3.1 Distributed parameter open loop control.

control error $E(x, k) = W(x, \infty) - Y(x, k)$ with quadratic norm reaches its minimum $\|\bar{E}(x, \infty)\|$ in the new steady-state of HLDS for $k \rightarrow \infty$:

$$\begin{aligned} \min \|E(x, \infty)\| &= \min \|W(x, \infty) - Y(x, \infty)\| = \\ &= \|W(x, \infty) - \bar{Y}(x, \infty)\| = \|\bar{E}(x, \infty)\| \end{aligned} \quad (3.1)$$

where $\bar{Y}(x, \infty)$ is the optimal output in the new steady-state.

First, in the block of space synthesis (SS), we solve the approximation problem on the linear space of distributed parameter quantities $\mathbf{X}$ in steady-state on $[0, L]$ with the quadratic norm (see Fig. 3.3), where it is assumed that $\{\mathcal{H}HR_i(x, \infty)\}$ forms the basis of the finite-dimensional linear subspace of the outputs of HLDS in steady-state, $\mathbf{Fn}$.

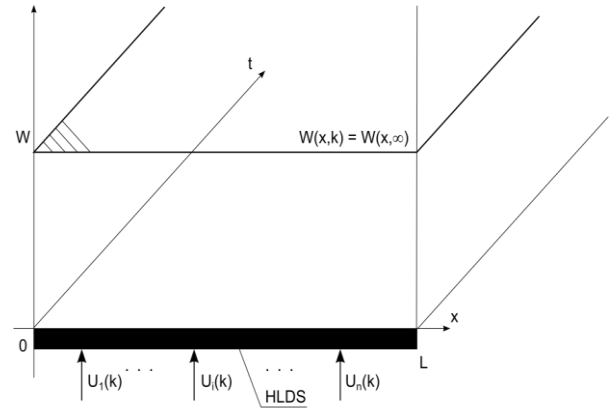


Fig. 3.2 Step change of the distributed reference $W(x, \infty)$.

$$\begin{aligned} \min \left\| W(x, \infty) - \sum_{i=1}^n W_i(x_i, \infty) \mathcal{H}HR_i(x, \infty) \right\| &= \\ = \left\| W(x, \infty) - \sum_{i=1}^n \bar{W}_i(x_i, \infty) \mathcal{H}HR_i(x, \infty) \right\| &= \\ = \|W(x, \infty) - \bar{W}O(x, \infty)\| \end{aligned} \quad (3.2)$$

Note, that $\bar{W}O(x, \infty)$ is the unique best approximation of $W(x, \infty)$ and $\bar{W}(\infty) = \{\bar{W}_i(x_i, \infty)\}$ are optimal

approximation parameters, furthermore, that the deviation of $W(x, \infty)$ and $\tilde{W}O(x, \infty)$ has a minimal quadratic norm.

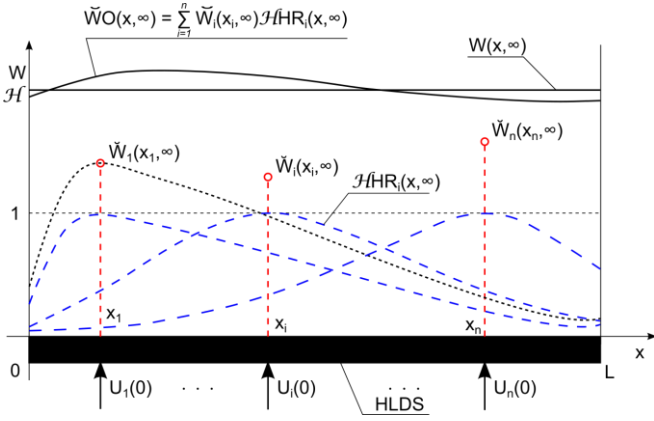


Fig. 3.3 Approximation task of reference quantity $W(x, \infty)$ in the block of space synthesis – SS.

The optimal approximation parameters $\tilde{W}(\infty) = \{\tilde{W}_i(x_i, \infty)\}$ now enter the time synthesis algorithmic block (TS) as reference (see Fig. 3.4). In this algorithmic unit we have n SISO control loops with plant and controller pairs $\{SH_i(x_i, z), R_i(z)\}$. The controlled systems in these units are respective HLDS time domain components of the dynamics: $\{SH_i(x_i, z)\}$. The controllers $\{R_i(z)\}$ are designed in a way that control errors

$$\{\tilde{E}_i(x_i, k)\} = \{\tilde{W}_i(x_i, \infty) - Y_i(x_i, k)\} = \tilde{\tilde{E}}_i(x_i, k) \quad (3.3)$$

reach zero steady-state in each SISO control loop in the new steady-state:

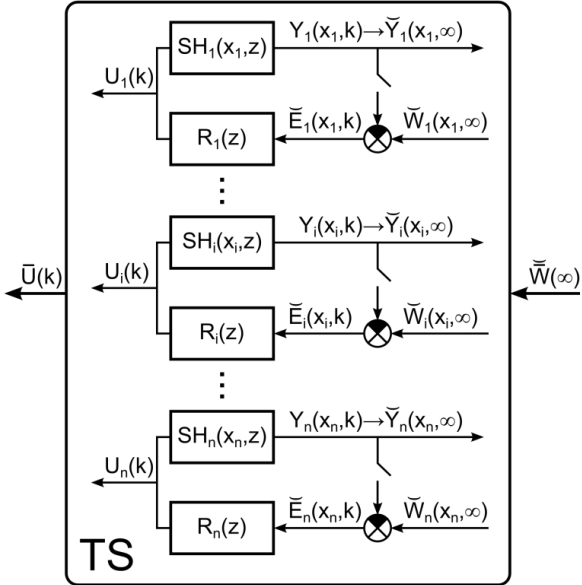


Fig. 3.4 SISO control loops in the time synthesis block.

$$\begin{aligned} \left\{ \lim_{k \rightarrow \infty} \tilde{E}_i(x_i, k) \right\} &= \left\{ \lim_{k \rightarrow \infty} [\tilde{W}_i(x_i, \infty) - Y_i(x_i, k)] \right\} = \\ &= \left\{ [\tilde{W}_i(x_i, \infty) - \tilde{Y}_i(x_i, \infty)] = 0 \right\} \end{aligned} \quad (3.4)$$

When the individual components of the vector $\tilde{W}(\infty) = \{\tilde{W}_i(x_i, \infty)\}$ act on respective SISO control loops $\{SH_i(x_i, z), R_i(z)\}$, the individual control processes take place. From this, we obtain sequences of inputs $\bar{U}(k) = \{U_i(k)\}$ which provide partial particular outputs through the $\{SH_i(x_i, z)\}$ blocks in SISO loops; converging to the respective lumped reference quantities for $k \rightarrow \infty$ $\{Y_i(x_i, k) \rightarrow \tilde{Y}_i(x_i, \infty) = \tilde{W}_i(x_i, \infty)\}$ (3.5)

When $\{U_i(k)\} = \bar{U}(k)$ will appear on the input of HLDS (through the $\{SH_i(x_i, z)\}$ blocks as temporal components of HLDS dynamics) we will have for $k \rightarrow \infty$

$$\{\tilde{W}_i(x_i, \infty) = \tilde{Y}_i(x_i, \infty)\} \quad (3.6)$$

in the new steady-state, (see Fig. 3.5).

Finally, this means that the distributed output quantity $\tilde{Y}(x, \infty)$ gives the best approximation of $W(x, \infty)$ in the new steady-state.

$$\begin{aligned} \tilde{Y}(x, \infty) &= \sum_{i=1}^n \tilde{Y}_i(x_i, \infty) \mathcal{H}HR_i(x, \infty) = \\ &= \sum_{i=1}^n \tilde{W}_i(x_i, \infty) \mathcal{H}HR_i(x, \infty) = \tilde{W}O(x, \infty) \end{aligned} \quad (3.7)$$

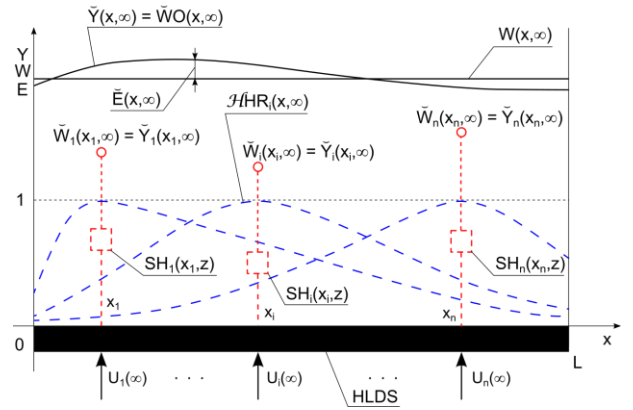


Fig. 3.5 $\{U_i(k) \rightarrow U_i(\infty)\}$ through $\{SH_i(x_i, z)\}$ setting up particular outputs into the new steady-states $\{\tilde{Y}_i(x_i, \infty)\}$.

Following the discussion above, a minimum-norm control error is also uniquely given by

$$\begin{aligned} \|\tilde{\tilde{E}}(x, \infty)\| &= \|W(x, \infty) - \tilde{Y}(x, \infty)\| = \\ &= \left\| W(x, \infty) - \sum_{i=1}^n \tilde{Y}_i(x_i, \infty) \mathcal{H}HR_i(x, \infty) \right\| = \\ &= \left\| W(x, \infty) - \sum_{i=1}^n \tilde{W}_i(x_i, \infty) \mathcal{H}HR_i(x, \infty) \right\| = \\ &= \|W(x, \infty) - \tilde{W}O(x, \infty)\| \end{aligned} \quad (3.8)$$

We may state that the distributed parameter control synthesis is decomposed into:

Temporal synthesis - by SISO control loops

Spatial synthesis - as approximation problem

IV. FEEDBACK CONTROL LOOPS

Based on the decomposition of dynamics and control synthesis, closed control loops are also assembled, Fig. 4.2-Fig. 4.4, [1-4, 11]. In control loops in Fig. 4.2 and Fig. 4.4 continuous controlled variables in sampling times $Y(x, k)$ or vectors of controlled variable values at the points of numerical model mesh nodes $\{Y(x_q, k)\}$ are considered. The control loop in Fig. 4.3 is based on the measurements of controlled quantity $\bar{Y}(x, k) = \{Y(x, k)\}$. Outputs of space synthesis blocks SS and SS1 give a control error $\bar{\bar{E}}_i(x, k) = \{\bar{E}_i(x, k)\}$ as in relation (3.3), which enters the TS block, Fig. 4.1, where the controllers are set by the relation (3.4), (see Fig. 3.4). If the steady-state control error vector is zero $\bar{\bar{E}}_i(x, \infty) = 0$ then the quadratic norm of the distributed control error $\|W(x, \infty) - Y(x, \infty)\|$ will be minimized $\|\bar{E}(x, \infty)\|$, thereby fulfilling the control task.

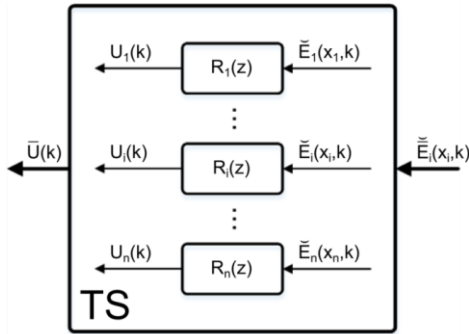


Fig. 4.1 Controllers of temporal synthesis.

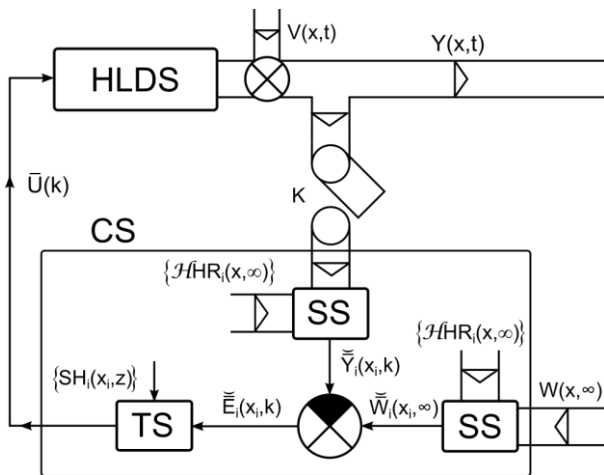


Fig. 4.2 Distributed parameter closed control loop.

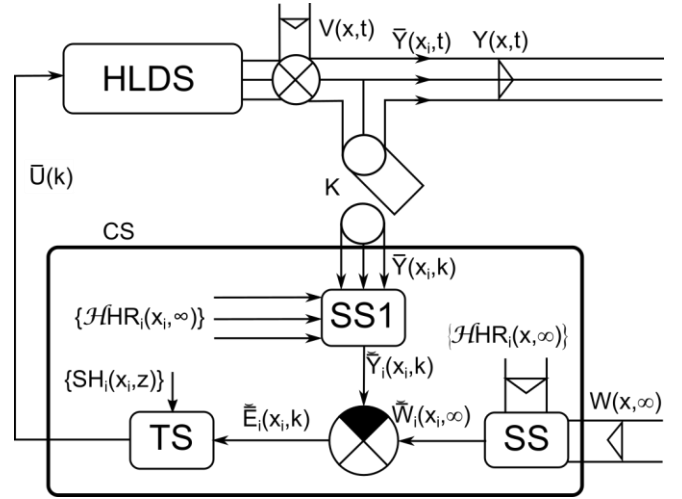


Fig. 4.3 Distributed parameter closed control loop with measurements in selected points of controlled object.

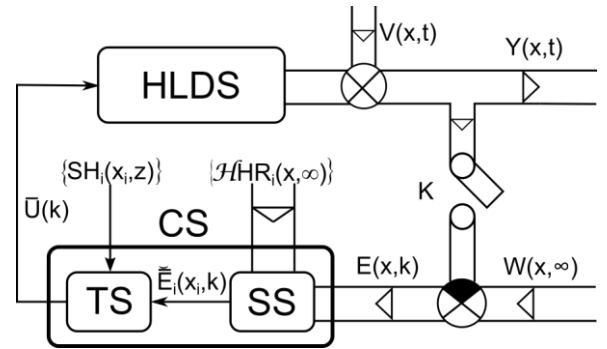


Fig. 4.4 Distributed parameter feedback control loop.

V. DPS BLOCKSET FOR SIMULINK

Based on introduced control loops for distributed parameter systems (DPS), a software support named DPS Blockset for Simulink is being developed in the framework of the CONNECTIONS program of The MathWorks Company. The DPS Blockset for Simulink is declared as third-party software product of The MathWorks Company, Fig. 5.1.

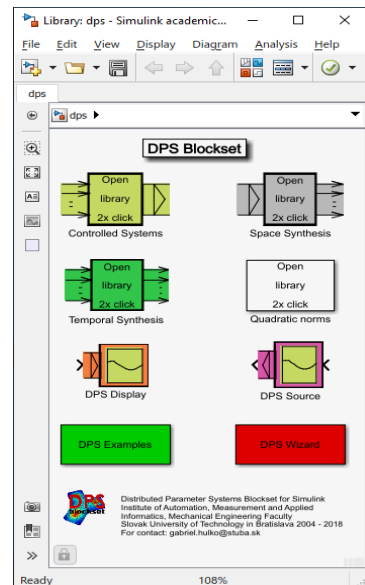


Fig. 5.1 Library of the DPS Blockset for Simulink.

The DPS Blockset for Simulink includes blocks for modeling the dynamics of controlled systems - HLDS, for the synthesis of control in spatial and temporal coordinates. For generation distributed references and display results of simulations blocks DPS Source and DPS Display are offered. Norms of lumped and distributed quantities are computed by block Quadratic norms. The block Examples presents some model control tasks. The DPS Wizard offers the possibility of automated design of distributed parameter control loops. Based on the DPS Wizard, the Interactive Control service is built to support the solution of distributed parameter system control tasks over the Internet, [3].

VI. CONTROL OF SECONDARY COOLING IN CONTINUOUS CASTING OF STEEL

In order to demonstrate the possibilities of the proposed engineering approach to control distributed parameter systems as well as DPS Blockset for Simulink software support, the control of secondary cooling process in continuous casting of steel around the specified steady-state operating mode is considered, [5, 10-12]. The secondary cooling process is a real NLDS, Fig. 6.1, where the lumped input quantities are the flow rates of the cooling waters $\{U_i(t)\}$ and the temperature field on the inner radius of the casting strand in the secondary cooling zone, in the linearized surrounding of the given operating mode, is the distributed controlled variable $Y(x, t)$. Linearized numerical model was built up in ProCAST. Typical distributed parameter step responses at given steady-state operating mode are shown in Fig. 6.2 as $\{\mathcal{H}_i(x, k)\}$.

The control loop is composed of the algorithmic blocks found in the DPS Blockset for Simulink, Fig. 6.3. Space synthesis blocks carry out the space synthesis approximation tasks. The control synthesis in the temporal relation is solved by algorithms of constrained MPC based on Kalman filter state estimator, [10].

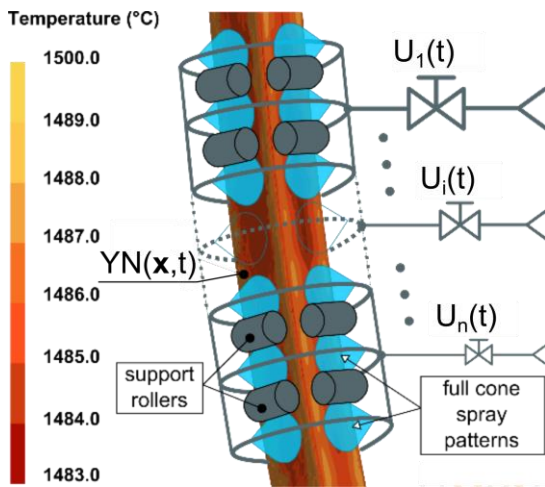


Fig. 6.1 Secondary cooling zone (SCZ) in process of continuous casting of steel.

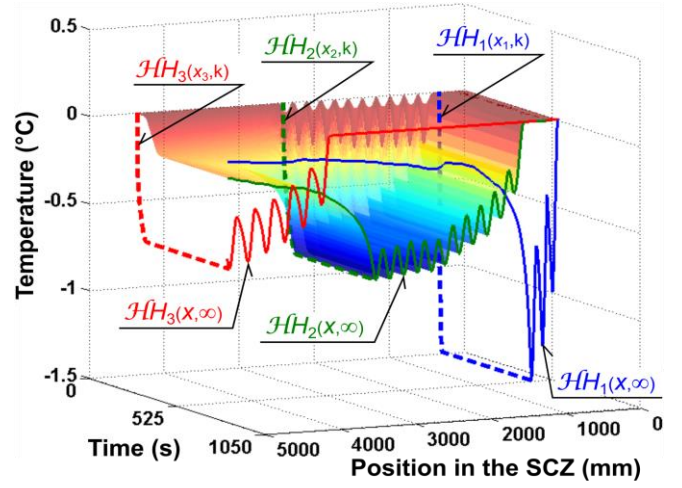


Fig. 6.2 Typical profiles of distributed parameter step responses on inner radius of secondary cooling zone.

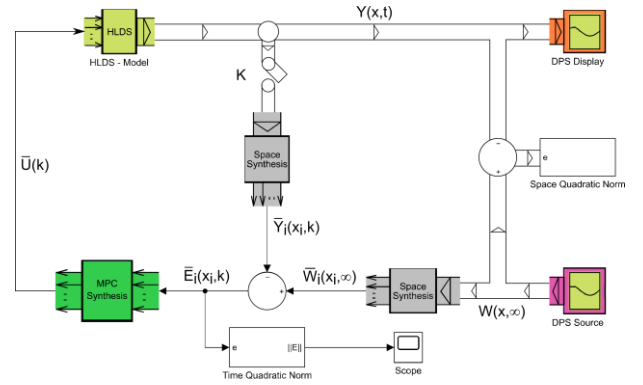


Fig. 6.3 Control loop for secondary cooling in continuous casting of steel.

Some applications of this engineering approach are also demonstrated in the areas of energy systems [6], induction heating [8], extrusion of plastics [7] and groundwater remediation [9].

Control task of the temperature field reference tracking, disturbance rejection and operating mode discrete changes are shown in Figs. 6.4 – 6.6.

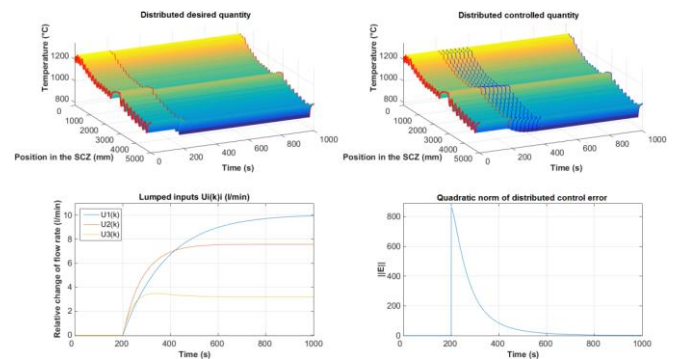


Fig. 6.4 Reference tracking: transition to a new temperature field at constant casting speed (0.90 m/min).

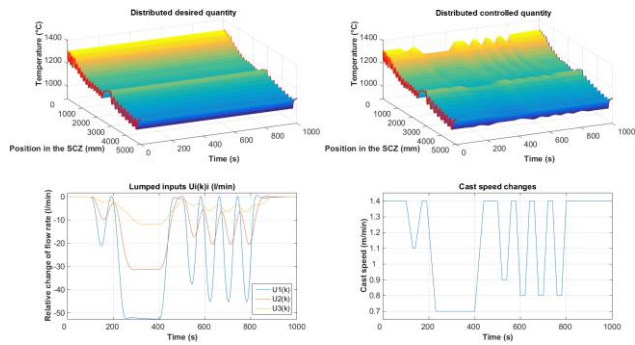


Fig. 6.5 Disturbance rejection: elimination of temperature field oscillation under casting speed abrupt changes (from 0.70 to 1.40 m/min).

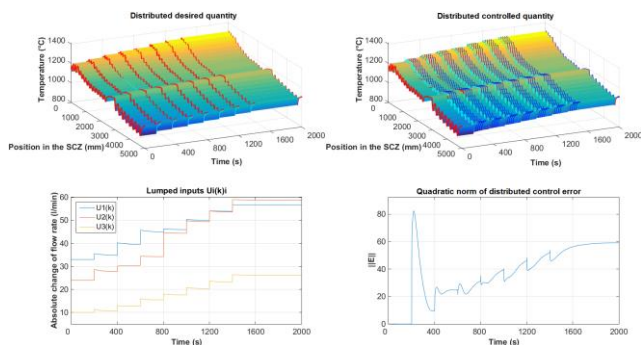


Fig. 6.6 Discrete changes of operating modes (according to casting speed changes) while following the reference temperature field.

VII. CONCLUSIONS

In reality, all technological and manufacturing processes are nonlinear distributed parameter systems. Their time-space dynamic characteristics are often investigated by simulations using numerical models. In this paper a control synthesis of distributed parameter systems based on these numerical models as lumped input and distributed parameter output systems has been proposed. Its practical utility should be leveraged by applied simulation studies, aimed on the selection and deployment of inputs and measurement points for space synthesis as well as the choice of methods for time synthesis.

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